Technical Notes

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Real-Gas Effect on the Magnetoplasmadynamic Arcjet

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Nomenclature

B = self-induced magnetic field
C_p = specific heat at constant pressure
E = electric field, energy level
F = mass flow rate per unit area
h = enthalpy per unit mass
J = total current
j = current density
k = Boltzmann constant
m = mass of particle
n = particle number density
p = pressure
R = gas constant
T = temperature
U_{ion} = ionization and dissociation energy per unit mass
u = velocity
V = volume
x = coordinate along the flow
Z = partition function
μ_0 = magnetic permeability
ρ = mass density
σ = electrical conductivity

Subscripts

e = electron
i = ion
0 = inlet

Introduction

THE magnetoplasmadynamic (MPD) arcjet thruster has been developed for future applications of interplanetary explorations and orbit transfer missions. From a practical point of view, molecular propellants such as ammonia, hydrazine, and hydrogen will be promising because they provide higher thrust efficiency than monatomic propellants.[1] One of the most interesting features of molecular propellants is that their discharge phenomena are quite different from those of monatomic propellants, Ar, for instance, which is extensively used in the MPD arcjet. Recent work[2] suggests that the discharge current in a molecular propellant concentrates down to the cathode tip while an Ar propellant uniformly discharges. Although many explanations have been offered to interpret this phenomenon incorporating induced magnetic fields and plasma interaction, none were a reasonable explanation.[2,3] This report describes that the essential process in molecular propellants must involve dissociation as well as ionization, and the inclusion of this process may consistently predict the MPD arcjet performances.

Basic Assumptions of Analysis

For simplicity, the governing equations in this analysis employ conventional magnetohydrodynamic (MHD) expressions whose validity should be discussed elsewhere.[4–6] The geometry is shown in Fig. 1. Our basic assumptions are 1) flow is one-dimensional and in the steady state, 2) compressibility is taken into account but not viscosity, and 3) plasma conductivity must vary with thermodynamic states.

The last assumption must play an important role in this context and, exactly speaking, most of the plasma flow cannot reach the complete local thermodynamic equilibrium (LTE) in the MPD arcjet but reaches the partial LTE in its higher energy levels.[7] In this sense, the rate equations of both dissociation and ionization may be used instead of equilibrium equations; however, the concrete process itself, how the plasma reaches its equilibrium, cannot be of interest here. We preferred physical simplicity rather than numerical fitness of our calculation to the experimental data, for which purposes those semiempirical rate equations are useful. The basic equations are as follows:

Mass conservation:

$$\frac{\mathrm{d}\rho u}{\mathrm{d}x} = 0 \tag{1}$$

Equation of motion:

$$\rho u \frac{\mathrm{d}u}{\mathrm{d}x} + \frac{\mathrm{d}p}{\mathrm{d}x} - jB = 0 \tag{2}$$

Equation of energy:

$$\rho u \frac{\mathrm{d}u^2/2}{\mathrm{d}x} + \rho u \frac{\mathrm{d}h}{\mathrm{d}x} - Ej = 0 \tag{3}$$

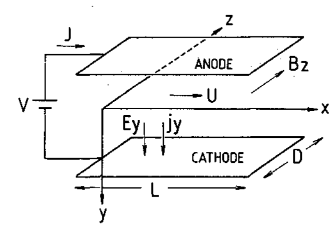

Fig. 1 One-dimensional flow model.

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Equation of state:

$$p = \rho RT \tag{4}$$

Ohm's law:

$$j = \sigma(E - uB) \tag{5}$$

Induction equation:

$$j = -\frac{1}{\mu_0}\frac{\mathrm{d}B}{\mathrm{d}x} \tag{6}$$

To treat hydrogen and argon as real gases associated with dissociation of molecules and ionization of atoms, a thermal equilibrium equation is oriented:

$$\frac{n_e n_i}{n} = 2\frac{Z_i}{Z}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}\exp\left(-\frac{Ei}{kT}\right) \tag{7}$$

Since the specific heat at constant pressure is a function of both temperature and pressure, the enthalpy varies not only with temperature but also with pressure:

$$h = \Sigma\left[nkT^2\left(\frac{\partial \ell n Z}{\partial T}\right)_V + nkTV\left(\frac{\partial \ell n Z}{\partial V}\right)_T + U_{\text{ion}}\right] \tag{8}$$

The electrical conductivity is a function of plasma number density and electron collision cross section of momentum transfer for atomic, ionic, and molecular species[8-9]:

$$\frac{1}{\sigma} = \frac{1}{\sigma_n} + \frac{1}{\sigma_i} \tag{9}$$

In this equation σ_n and σ_i represent Chapman-Cowling and Spitzer-Harm formulation, respectively.[10]

The state variables are plasma density ρ, velocity of flow u, gas pressure p, temperature T, enthalpy h, current j, and self-induced magnetic field B. The magnetic field at the inlet B_0, mass flow rate per unit area F_0, temperature T_0, and Mach number M_0 at the inlet are given as the initial conditions. The magnetic field strength is proportional to the total current per unit width, $B_0 = \mu_0 J$. Furthermore, the flow is assumed subsonic at the inlet and supersonic at the exit.

Analytical Results

These equations are solved as a function of nondimensionalized magnetic field B/B_0 without discussing "cold-inlet difficulty" as seen in the combustion flows, where the zero temperature at the inlet cannot initiate the dissociation and ionization processes. Here, we assumed 1000 K at the inlet, which in turn leads to small but not zero degree of ionization for current conduction. By using B/B_0 expressions instead of geometrical length x, we can avoid the useless display of very thin current-density region.

The most promising result is obtained in the current distributions for argon and hydrogen exhibited in Fig. 2. Along the flow, the current monotonically increases in the case of hydrogen, whereas it concentrates near the inlet and the exit in the case of argon. These results agree qualitatively with the experimental observation.[7] The difference in current distribution patterns can be explained as follows. The electron number density of argon plasma is fully developed even near the gas inlet, whereas that of hydrogen plasma increases after completion of dissociation (Fig. 3). Since the electrical conductivity is strongly dependent on the electron number density in the weakly ionized plasma, the current patterns are dominated by how many degrees the propellant is ionized along the flowfield. Another feature of real-gas effects is the predicted temperature (Fig. 3). Inclusion of dissociation and ionization lowers the plasma temperature to a reasonable level of about 1 eV.

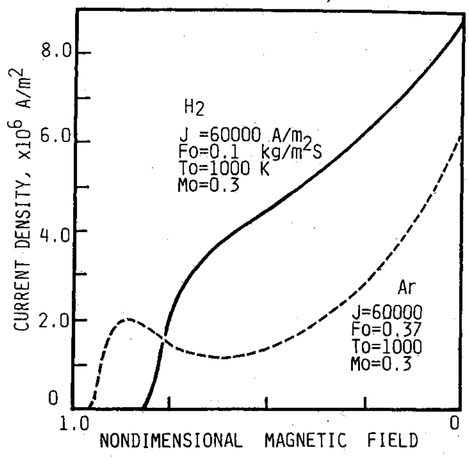

Fig. 2 Discharge current distributions for Ar and H_2 propellants.

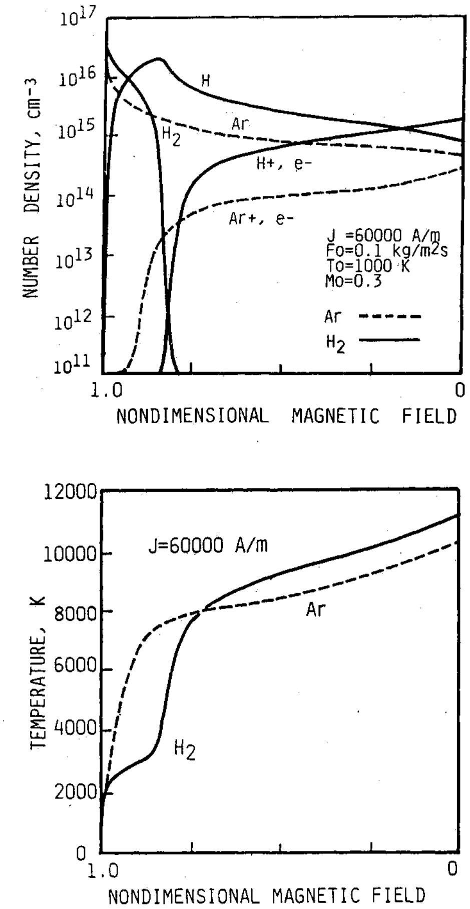

Fig. 3 Species number density (upper) and temperature distributions (lower) for Ar and H_2 propellants.

In the case of argon, the temperature shows rapid increase near the inlet; but in hydrogen, it indicates a plateau at about 0.3 eV, which corresponds to the dissociation temperature of that propellant. In this operating regime, gas static pressure is far less than 1 atm for the hydrogen propellant, and the formation of molecular ions such as H_2^+ may well be discarded from reaction processes. Likewise, the formation of negative ions H^- is not taken into account because the electron attachment is unlikely to occur except for the quiescent plasma.

Conclusions

The process including dissociation/ionization explains well the behavior of a molecular propellant in a self-field MPD arcjet. Before creating magnetized plasma, the discharge current must dissociate the molecular propellant, and during that process the plasma remains at low temperature corresponding to its dissociative energy absorption. As a consequence, most of the discharge current flows downstream region when the molecular propellant is used.

Acknowledgment

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Sensitivity of Shock/Shock Interactions to Upstream Variations

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Introduction

CURRENT plans for a transatmospheric vehicle, such as has been proposed for the National Aerospace Plane (NASP), depend on the design of weight-saving engine-integrated air-frames, in which the external vehicle surfaces act as the engine compression surface, and the aftbody would serve as the engine nozzle. It is generally assumed that the inlet will be designed so that its bow shock just contacts the cowl lip of the engine on design, although operating at this condition in steady state may not be possible.[1] One type of shock interaction that may occur when the bow shock contacts the shock formed on the engine cowl is known as the type IV shock interaction.

The type IV shock/shock interaction may result in localized regions of extremely high heating rates on the cowl of a hypersonic air-breathing engine. This work examines the effect that upstream variations have on the shock/shock interaction which results when a vehicle's inlet bow shock intersects with the engine's cowl bow shock. It is shown that slight perturbations in upstream conditions can have large effects on the type IV shock/shock interaction flowfield. The sensitivity of the flowfield to changes in various upstream parameters is presented, from which corresponding design rules for hypersonic inlets can be developed.

Previous analytical, experimental, and computational work has addressed the problem of shock/shock interactions.[2–9] The work presented here is an analytical study that considers how changes in freestream conditions, as described by the Mach number, and changes in the vehicle geometry, as described by changes in the vehicle bow shock deflection angle, influence the angle of the transmitted shock. This transmitted shock is a result of the vehicle's inlet bow shock intersecting the engine cowl bow shock, as shown in Fig. 1. This is of interest because, if the transmitted shock angle is strongly influenced by small changes in the upstream conditions, then it is likely that the entire interaction region will also be affected, along with the point of maximum heating on the cowl lip.

Results

In order to assess the significance of each upstream flow variable on the type IV interaction flowfield, a parametric study was done by solving the two-dimensional Rankine-Hugoniot relations. The effect of changes in freestream Mach number M_∞, incident shock angle β_i, and the cowl bow shock angle β_b, on the transmitted shock angle β_t were studied. The analytical approach has been to examine the sensitivity of the transmitted shock angle to changes in one parameter while holding the other two parameters constant.

Transmitted Bow Shock Angle Versus Mach Number

Varying Cowl Bow Shock Angles

For this study a constant inlet deflection angle of 10 deg was assumed. The results are shown in Fig. 2a. It is seen from

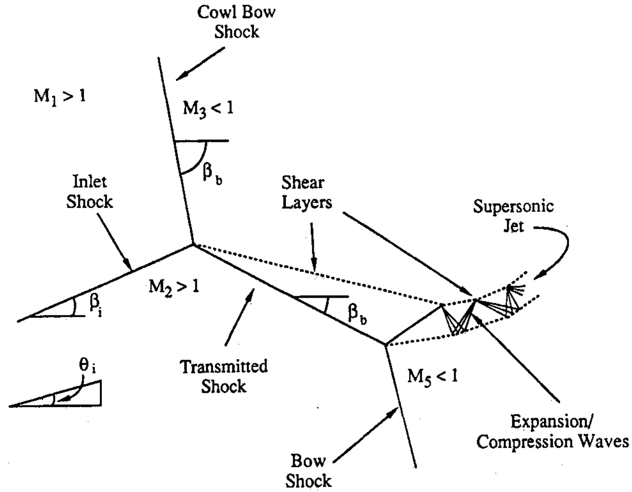

Fig. 1 Diagram of the type IV interaction.

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